

Partition method for improvement of piecewise linear model with full envelope covered

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Abstract: Since ambient conditions vary in a wide range within the full flight envelope, the existing piecewise linear model (PLM), which is based on sea-level static condition with use of corrected parameters for other points in the flight envelope, cannot meet the accuracy for replacing the nonlinear model. To obtain more accurate linear models, a method of partitioning the flight envelope over a grid of Mach number and altitude boxes was suggested. Then, a set of linear models for a given operating condition was selected by picking the nearest (Mach number, altitude) box in the flight envelope. Through the selected set of linear models, interpolating for power level based on a weighted sum of corrected rotor speeds can obtain a linear model with acceptable accuracy. Simulation results of different points within the full flight envelope showed that the maximum error between nonlinear model and the existing PLM was more than 50%, while the maximum error between nonlinear model and the improved PLM was within 8%. It is concluded that the improved PLM performs accurately, especially under the non-standard conditions. In addition, the improved PLM can satisfy the real-time requirement better than the existing PLM.

Key words: piecewise linear model; partition; full envelope; parameter schedule; turbofan engine

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Nomenclature

n_{lc}	percent corrected high-pressure-rotor speed	V_0	engine inlet air flow velocity
n_{fc}	percent corrected fan speed	V_9	nozzle exit gas flow velocity
F	thrust of the engine	H	altitude
W_a	engine intake air mass flow	H_{marked}	altitude of chosen condition
W_f	fuel mass flow	Ma	flight Mach number
W_{core}	core air mass flow	Ma_0	flow Mach number at inlet section
W_{duct}	bypass air mass flow	Ma_9	flow Mach number at nozzle exit
p_0	engine inlet total pressure	Ma_{marked}	flight Mach number of chosen condition
p_{3s}	compressor outlet static pressure	$\tau(Ma)$	aerodynamic function
T	total temperature	k	gas adiabatic coefficient
T_s	static temperature	R	gas constant
T_0	engine inlet total temperature	B	bypass ratio
T_{0s}	engine inlet static temperature	α	weighting factor to balancing n_{lc} and n_{fc}
T_9	nozzle exit total temperature	ρ	distance between two conditions
T_{9s}	nozzle exit static temperature	K_{ssp}	value of state schedule parameter
T_{se}	ambient temperature difference	K_{sn}	serial number of optimal point
V	velocity	K_i	serial number of selected row

K_j serial number of selected column

M number of columns

With the development of aero-engine design technology, fault diagnosis and health management increasingly attracted widespread attention^[1-4]. In general, fault diagnosis and health management require the airborne model with accuracy as well as high execution speed^[5-8]. In terms of the aero-engine modeling, although the nonlinear thermodynamic models are the most exact, linear dynamic models are still needed to exercise controller hardware during system integration. Linear models are different from the nonlinear models. They are computationally faster, thus able to operate in real time. However, the above merit also bring about a shortcoming that linear models could only be used in a very narrow range of the operating conditions^[9-10]. In order to extend the range and make the linear models available in the entire area of flight envelope, these key challenges should be faced:

1) Get several linear models to ensure that all operating conditions in the flight envelope are covered.

2) Match the current operating condition with the proper linear models at any time^[11].

3) Switch from one linear model to another smoothly while the operating condition is varying^[12-13].

For the first challenge, the existing method suggests that classifying all conditions to two types: standard sea-level conditions and other conditions. Under the sea-level conditions, operating points are chosen from idle to full power. For example, power range can be defined by percent corrected high-pressure-rotor speed n_{2c} from 70% to 100%, where $n_{2c} = 70\%$ represent idle and $n_{2c} = 100\%$ represent full power. To cover this power range, 7 operating points could be selected of which the values are $n_{2c} = 70\%$, 75%, 80%, ..., 100%. As for the other conditions, the existing method suggests that reducing the engine parameters to the sea-level conditions by the similarity principle.

The second challenge is about the problem

of scheduling linear models. While the engine model is running, there is a need to select the proper linear models at any time. The states of selected linear models should be close to the current operating states so that the results could be more accurate. To describe how close it is between two states, a special parameter is defined as state scheduling parameter (SSP). Practically, n_{2c} is often used as SSP for convenience. In the case of commercial engines with big fans, percent corrected fan speed n_{1c} is commonly used as SSP^[14]. Moreover, SSP could be considered as a function of all states^[15].

Since the third challenge is not the focus of this paper, no expansion will be followed.

To deal with the challenges above, piecewise linear model (PLM) in reference^[11], with quite simply structure and high execution speed, is the common approach in engineering. PLM has been widely used^[16-17], however, the accuracy of the existing PLM is unacceptable under the non-standard conditions. The reason will be analyzed and the simulation result will be displayed in later sections. To overcome the issue, this paper devotes to proposing a partition method to improve the existing PLM, and the simulation results verify that the accuracy of the improved PLM is significantly increased.

1 Existing PLM

To make the linear models available in the entire area of flight envelope, PLM has been proposed. PLM suggests extracting linear models at different operating points under standard sea-level conditions, and scheduling them according to the SSP. Overall, the quantity of operating points determines the accuracy of PLM. More points means more accuracy but slower computing speed. If the actual operating condition is non-standard, PLM will still schedule linear models in the same way, just using the reduced parameters instead. In a word, the essence of the existing PLM is scheduling linear

models under standard sea-level conditions.

As for reduced parameters, however, the similarity principle cannot be applied in every situation. The similarity principle requires a fixed flight Mach number. When the flight Mach number changed, some similar parameters lost their similarities. For instance, the thrust similar parameter expression is the following formula

$$\frac{F}{p_0} = \frac{W_a \sqrt{T_0}}{p_0} \left[\sqrt{\frac{T_9}{T_0}} \cdot \frac{V_9}{\sqrt{T_9}} - \frac{V_0}{\sqrt{T_0}} \right] \quad (1)$$

Where subscript 0 means the inlet section, and subscript 9 means the exit of nozzle section. According to gas dynamics

$$\begin{cases} V_9 = Ma_9 \sqrt{kRT_{9s}} \\ V_0 = Ma_0 \sqrt{kRT_{0s}} \\ \frac{T_s}{T} = \tau(Ma) \end{cases} \quad (2)$$

Where subscript s of temperature means static parameters, no subscript s means total parameters. The formula (1) can be transformed to the following formation (assume the gas adiabatic coefficient k is a constant)

$$\frac{F}{p_0} = \frac{W_a \sqrt{T_0}}{p_0} \left[\sqrt{\frac{T_9}{T_0}} f(Ma_9) - f(Ma_0) \right] \quad (3)$$

It is obvious that the thrust similar parameter depends on not only corrected mass flow but also Mach number.

When the flight Mach number keeps constant, the speed similar parameter can use the formula (3) to ensure that the thrust similar parameter is correct. On the contrary, the changing flight Mach number will cause error in parameter calculation because the similarity of the air flow in each section of the engine has been broken. Therefore, it is uncertain that the physical parameters in engine through conversion under the similarity principle are precise in the entire area of flight envelope.

Referred to formula (2), keep the Mach number stable, and the gas adiabatic coefficient is changed accompany with the temperature, therefore, the similar parameter $V/\sqrt{T}$ cannot keep constant at every altitude. On the other hand, the parameter reduction error as referred

before will become larger because of the variable specific heat calculation in engine nonlinear model^[18].

In the aspect of reducing computation and saving memory, applying similarity principle to extend the range of the PLM from standard condition to non-standard conditions maybe reasonable. However, in the actual test, the error it caused has beyond acceptable range.

2 A partition method for non-standard conditions

Known that using reduced parameters to the existing PLM under the non-standard conditions may not meet the accuracy requirement, especially when the actual condition is far from the standard sea-level condition. To describe the relationship between different conditions, a concept of distance ρ can be defined as

$$\rho = \sqrt{\sum_{i=1}^n w_i (X_i - Y_i)^2} \quad (4)$$

Where X and Y are two conditions, they both have n parameters. And $w_i (i=1, \dots, n)$ are the weighting factors depending on the importance of condition parameter. Since the atmosphere conditions can be determined by altitude and Mach number (the same as inlet temperature T and pressure p), the n in formula (4) is 2, and formula (4) can be simplified as

$$\rho = \sqrt{w_1 (X_1 - Y_1)^2 + w_2 (X_2 - Y_2)^2} \quad (5)$$

Where subscript 1 is of the altitude while subscript 2 is of the Mach number. In order to make the distance easy to understand, a two-dimensional coordinate system is established, one axis represents altitude H while the other axis represents Mach number Ma . Then a point can be used to describe an operating condition. And when w_1 and w_2 are equals to 1, Euclidean distance between two points is just the distance we defined between two conditions.

To avoid using the similarity principle in such a large range, this paper proposes an improvement for the existing PLM, based on a partition method.

PLM should take the non-standard condi-

tions into consideration by adding non-standard linear models instead of simply using reduced parameters. When breaking the entire area of flight envelope into a collection of range-limited areas, a set of points (Ma , H) can be used to describe the divided areas, and each single area corresponds to one point within the area. These points are defined as reference points. The new added linear models are constructed at every reference point, which means that each divided area corresponds to a set of linear models (from idle to full power at one condition) at its reference point. Compared with the improved PLM, the existing PLM that simply uses reduced parameters just has one area, corresponding to one set of linear models at reference point (0, 0 km), as shown in Fig. 1.

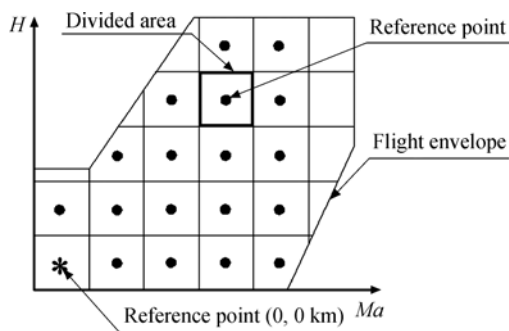


Fig. 1 Comparison of the existing PLM and the improved PLM

It's clear that the existing PLM just has models at reference point (0, 0 km), while improved PLM has models at every reference points. After partition processing, every reasonable point (condition) can be located in divided range-limited areas that contain reference points. It is noted that distance between the point and the corresponding reference point is not very far away, reduced parameters can be range-limited used as an approximate method in every divided area.

For example, area can be partitioned with 1 km altitude intervals and Mach number intervals of 0.1. As for a particular point (0.48, 5.1 km), reference point (0.5, 5 km) could be got according to the area in which they are both lo-

cated, as shown at Fig. 2. Then the parameters are reduced from the particular point (0.48, 5.1 km) to the reference point (0.5, 5 km). After that, linear models are scheduled at reference point by interpolation, just like the way to schedule the existing PLM.

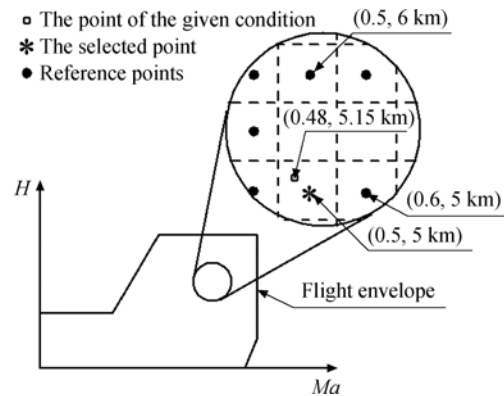


Fig. 2 Example of a given condition (0.48, 5.15 km)

The interpolation for the steady-state parameters and the matrix of linear model coefficients is according to the SSP. For a two spools turbofan, the SSP can be defined as following formula:

$$K_{ssp} = \alpha n_{1c} + (1 - \alpha) n_{2c} \quad (6)$$

Where K_{ssp} is the value of the SSP, α is the weighting factor depending on bypass ratio.

Essentially, α is the weight showing whether n_{1c} or n_{2c} is the main factor to represent the state of the engine. For the engine with lower bypass ratio, n_{2c} is the major factor to influence the engine state. When it turns to high bypass ratio, n_{1c} stands out as the major one. Therefore, α could be express as

$$\alpha = \frac{W_{duct}}{W_{duct} + W_{core}} = \frac{1}{1 + W_{core}/W_{duct}} = \frac{1}{1 + 1/B} \quad (7)$$

Where B is bypass ratio, W_{core} is the core air flow while W_{duct} is the bypass air flow.

In summary, the algorithm of the existing PLM contains three steps.

1) To reduce the parameters from any condition to the standard sea-level condition.

2) To calculate the SSP which is weighted by corrected rotor speeds n_{1c} and n_{2c} .

3) To interpolate based on the SSP under the standard condition (reference point at (0, 0 km)).

While the algorithm of the improved PLM contains four steps.

1) To locate which condition is the nearest condition according to altitude and Mach number.

2) To reduce the parameters from any condition to the nearest condition picked in step one.

3) To calculate the SSP which is weighted by corrected rotor speeds n_{1c} and n_{2c} .

4) To interpolate based on the SSP under the condition picked in step one.

As can be seen, the existing PLM schedules linear models by the SSP, while the improved PLM schedules linear models by (Ma , H) and the SSP as well.

3 Scheduling method of the improved PLM

The essence of scheduling linear models is scheduling sets of data. Known that PLM needs two types of data, one type is for static performance represented by static data; the other is for dynamic performance represented by matrices of PLM coefficients. In terms of scheduling method, however, there is no essential difference between the two types of data. The main challenge is determining how to select the proper data according to a particular condition.

While proposing a method, not only detail strategy should be described, but also the feasibility in all aspects should be analyzed.

3.1 Strategy A

For any particular condition, a point can be used to describe it, here it is named “marked point” for convenience. The marked point is in the same coordinate system as the reference points. Since all the reference points have been prepared, the directly way to pick up the proper one is checking them one by one, using a variable to record the shortest distance between the marked point and every reference point, and using another variable to record which reference point breaks the distance record newly. After a

traverse, the optimal reference point will be got, the distance between this point and the marked point is the shortest which means the most similar conditions. The strategy A is shown in Fig. 3.

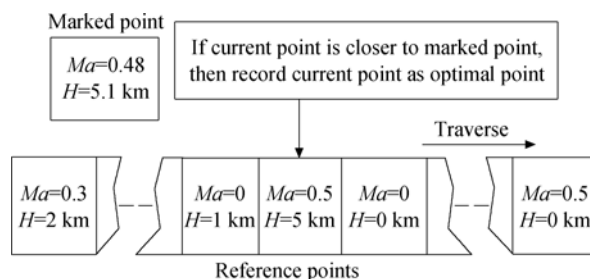


Fig. 3 Strategy A

3.2 Strategy B

To select the optimal proper point among the reference points, it seems unnecessary traversing all reference points. Organizing data in order is helpful to select the proper point efficiently. Inspired by the correlation between two-dimensional array and one-dimensional array, the reference points in a two-dimensional table are organized. Every row has the same number of elements, and rows are sorted by altitude. The elements in each row have the same altitude, and sorted by Mach number. Now, the locating algorithm is defines as follows.

1) For any point, the altitude is H_{marked} and Mach number is Ma_{marked} . It can be got that H_{marked} is the closest to the K_i -th height by one-dimensional interpolation;

2) In the K_i -th row, It also can be got that the Ma_{marked} is the closest to the K_j -th Mach number by one-dimensional interpolation;

3) If the number of elements in a row is M , the serial number K_{sn} of the optimal point can be calculated as follows:

$$K_{\text{sn}} = (K_i - 1)M + K_j \quad (8)$$

For instance, reference points have been prepared in Table 1 ($M=3$), and the marked point for current condition is (0.48, 5.1 km). According to the locating algorithm, K_i equals 2 because the number 5.1 closest to 5. Likewise, K_j equals 2. At last, the serial number of the selected reference point is 5.

Table 1 Sorted data

K_i	H/km	Ma			K_{sm}		
		$K_j=1$	$K_j=2$	$K_j=3$	$K_j=1$	$K_j=2$	$K_j=3$
1	0	0	0.2	0.4	1	2	3
2	5	0.3	0.5	0.7	4	5	6
3	10	0.5	0.6	0.7	7	8	9

3.3 Analysis of memory consumption

Clearly, there is no difference between the two strategies in terms of memory consumption. They both need to store data of all reference points, and the number of reference points depends on partition plan.

Although the improved PLM contains more data, the memory consumption is totally acceptable. As the result of practical test, one set of linear models require no more than 50 kB of memory. Assuming that N segments on the altitude axis, M segments on the Mach number axis, the total memory consumption of the improved PLM is about $50N \cdot M$ kB. Obviously, there is no need to care about the issue of memory consumption.

3.4 Analysis of algorithm complexity

Computing speed, related to the real-time performance, is quite more critical than memory consumption. Clearly, computing speed is decided by the complexity of algorithm.

In terms of strategy A: this algorithm is simple and easy to understand, but have to traverse all reference points at every step. Assuming that N segments on the altitude axis, M segments on the Mach number axis, this algorithm takes time $O(N \cdot M)$.

In terms of strategy B: according to the processes, the row with the nearest altitude is located by one-dimensional interpolation. The worst-case takes time $O(N)$, and it will take time $O(\log_2 N)$ using dichotomy method. Then the point with the nearest Mach number is got in the located row the same way which takes time $O(\log_2 M)$. Therefore, this algorithm takes time $O(\log_2 (N \cdot M))$.

Obviously, strategy B is better choice.

After getting serial number of the selected reference point, the improved PLM could be calculated just the same way as the existing PLM.

4 Simulate validation of the method

This section is to test the existing PLM and the improved PLM by simulation, and compare them with the nonlinear model. Here is the top-level structure of the simulation platform as shown in Fig. 4.

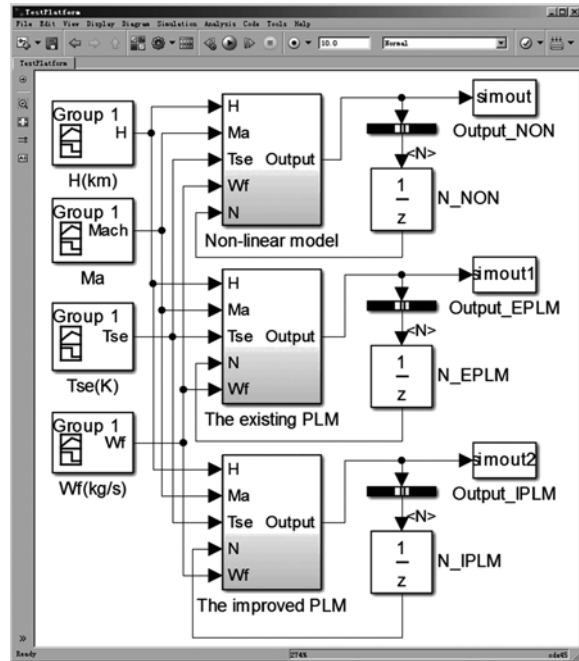


Fig. 4 Top-level structure of the simulation platform

From top to bottom in Fig. 4 are nonlinear model, the existing PLM and the improved PLM. Three models have same input signals: H , Ma , T_{se} (temperature difference between the sea-level local and the sea-level standard conditions) and the normalized fuel flow W_f . It can be considered that T_{se} always equals zero in this tests, which means the temperature at zero height is 288.15 K. The output data of the three models mainly contains the normalized static pressure of high compressor outlet p_{3s} , normalized thrust F , normalized EPR (engine pressure ratio, π), normalized EET (engine exhaust temperature, T_{ee}), n_{1c} and n_{2c} , which will be plotted for comparison.

A part structure of the existing PLM is shown in Fig. 5 and a part structure of the improved PLM is shown in Fig. 6.

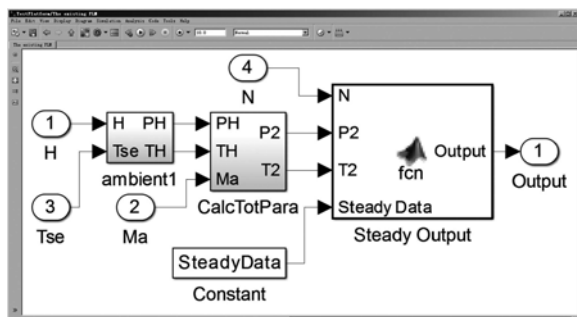


Fig. 5 Module for steady output of the existing PLM

In Fig. 6, the “scheduling method” module is designed for calculating the serial number that represents which one is the most appropriate reference point.

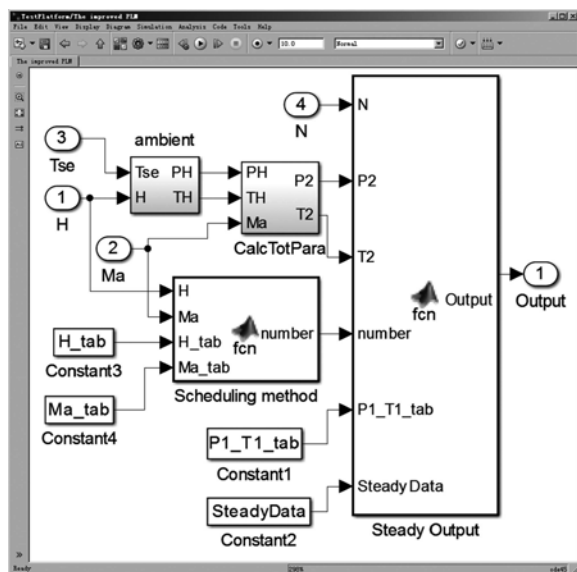


Fig. 6 Module for steady output of the improved PLM

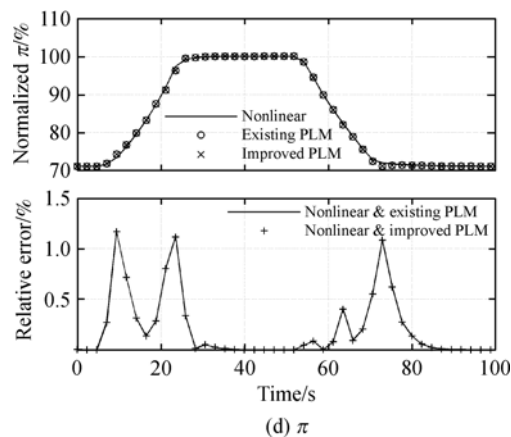
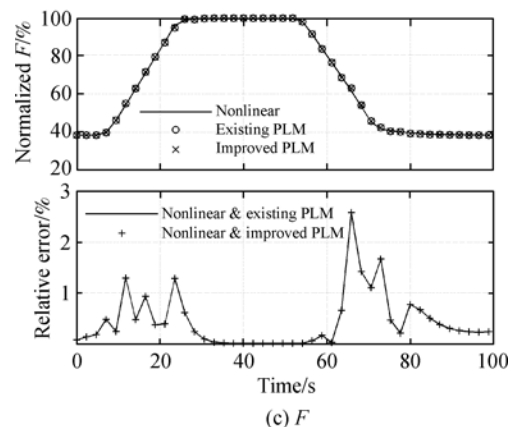
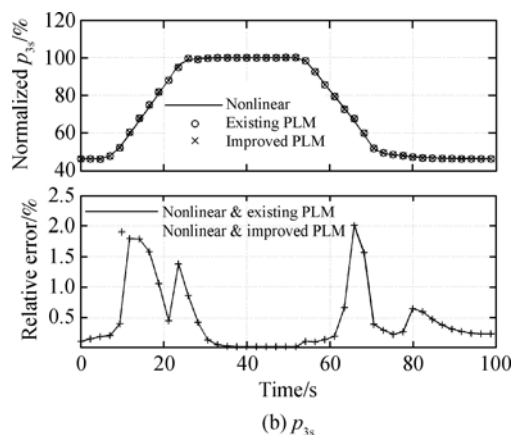
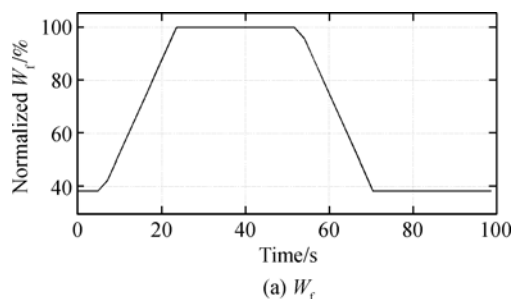
The specific plan of the full-envelope tests is listed in Table 2.

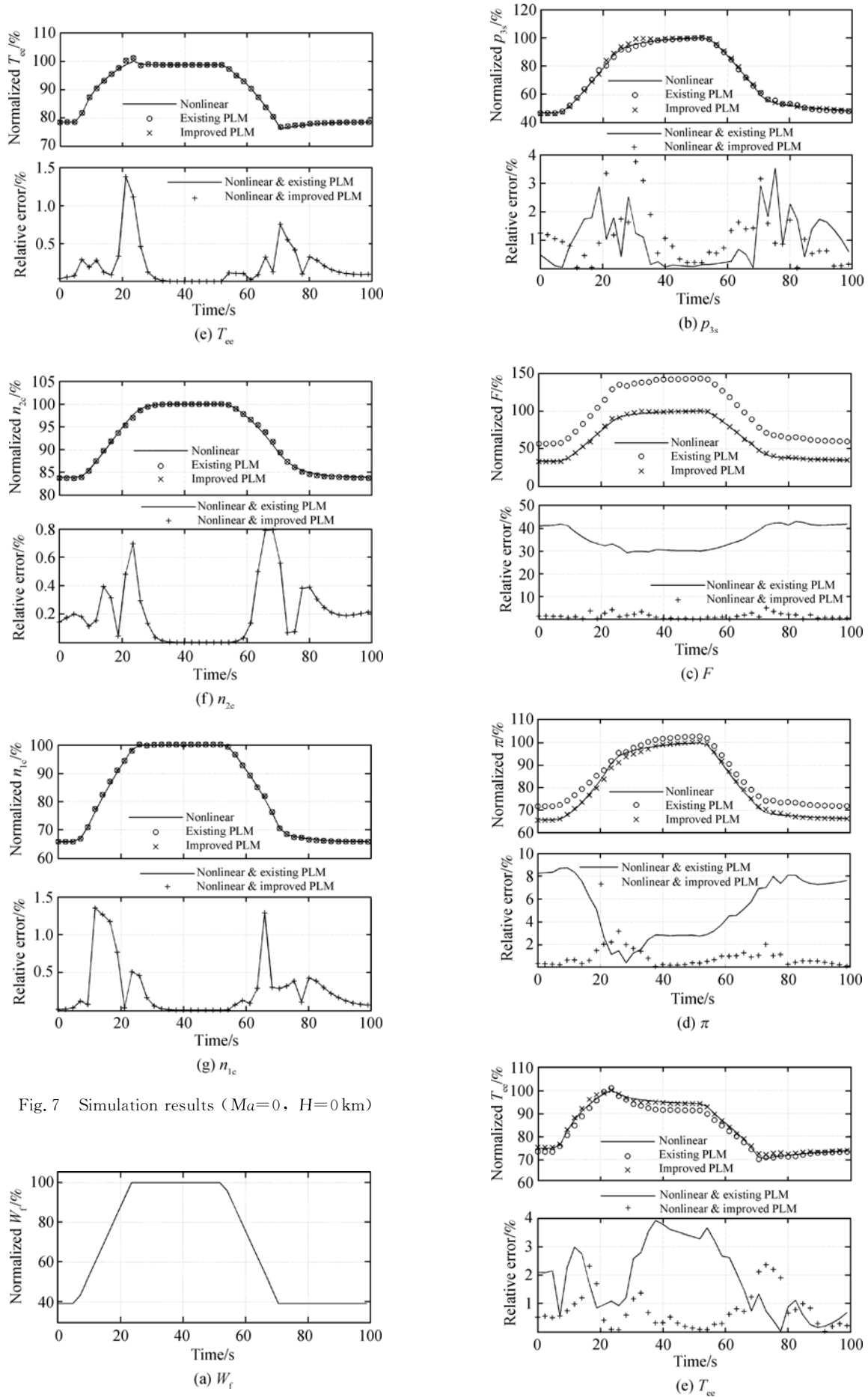
Table 2 Plan of the full-envelope tests

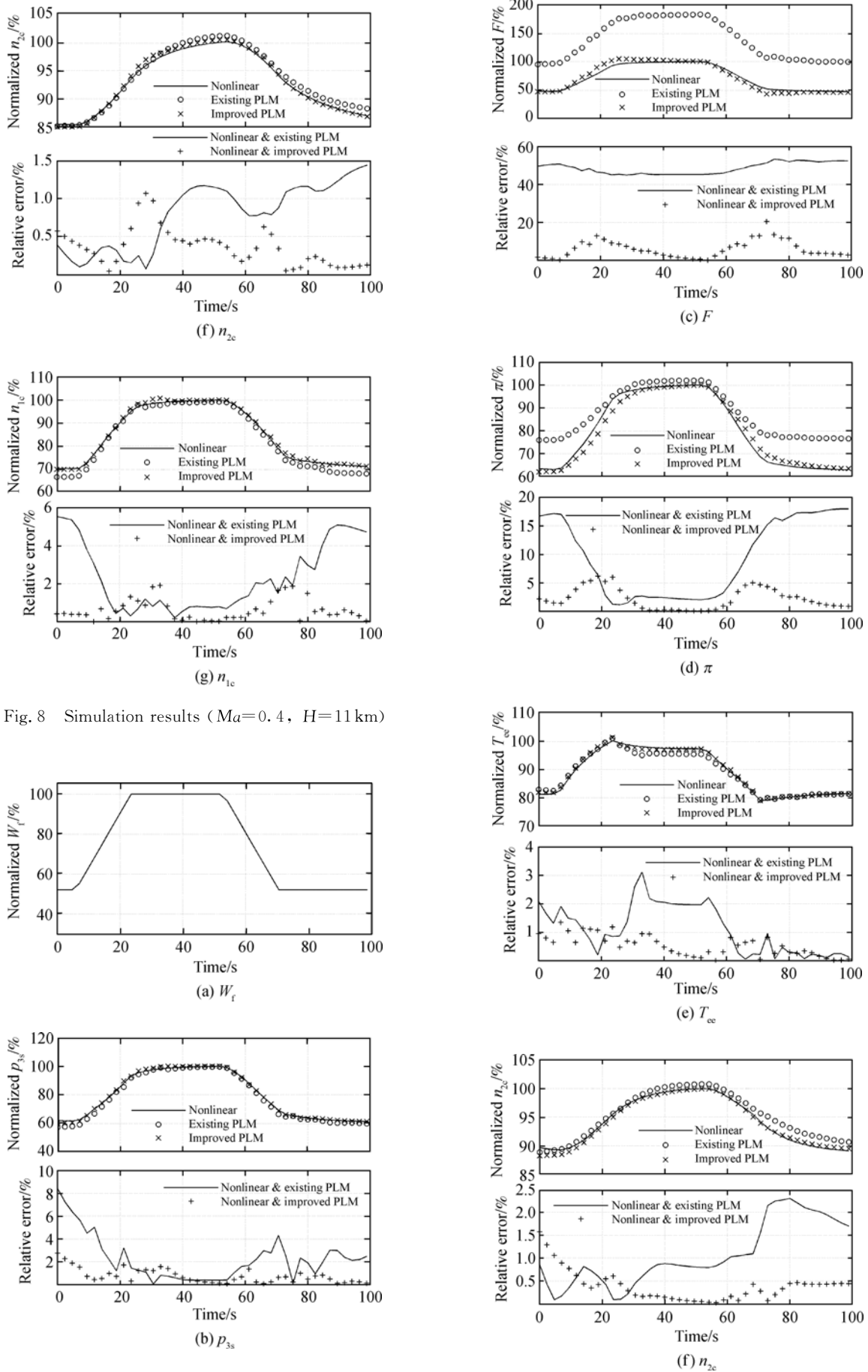
No.	H/km	Ma	T_{se}/K	Fuel flow
1	0	0	0	ramp from idle to
2	11	0.4	0	full power, then
3	11	0.8	0	ramp from full
4	0	0.5	0	power to idle

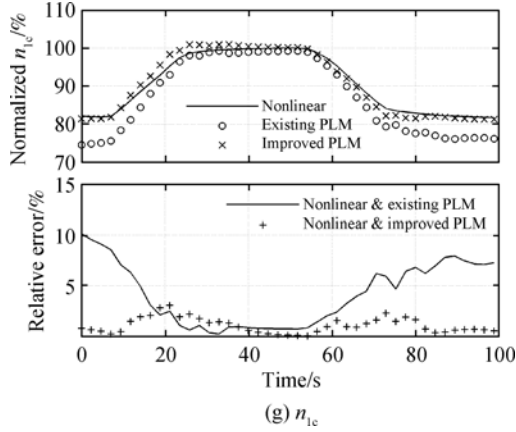
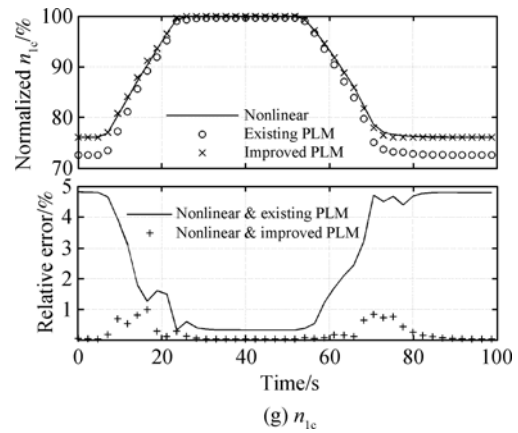
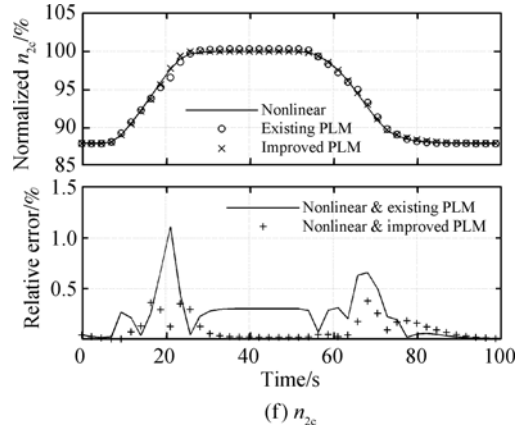
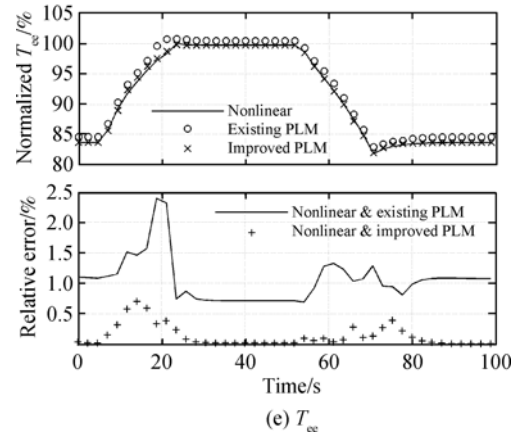
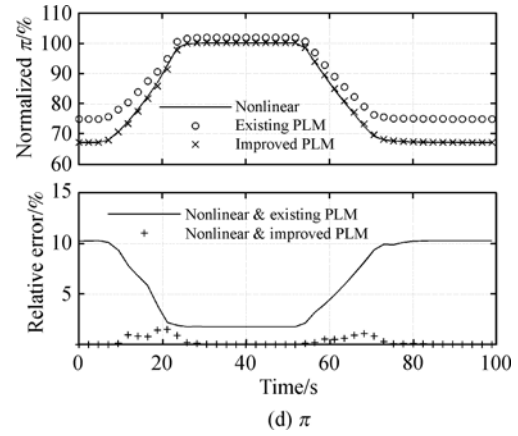
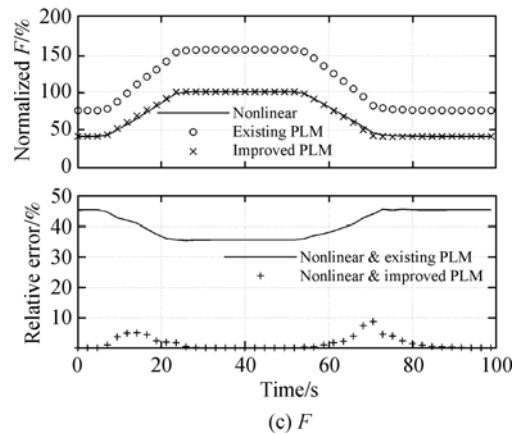
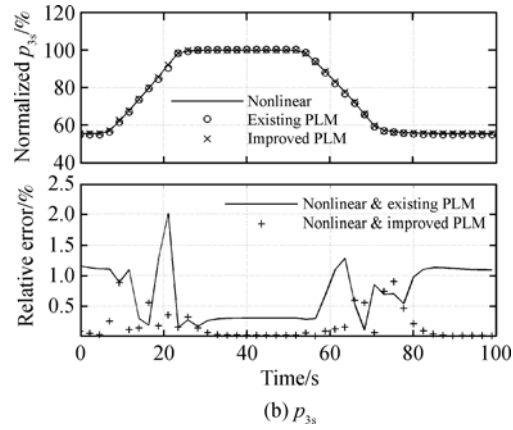
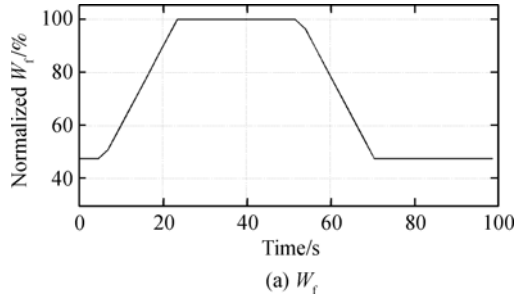
In addition, the simulation results are shown in Fig. 7–Fig. 10.

Figure 7 illustrates the simulation results under $H=0\text{ km}$, $Ma=0$. Fig. 7(a) shows the in-





Fig. 8 Simulation results ($Ma=0.4$, $H=11$ km)

Fig. 9 Simulation results ($Ma=0.8$, $H=11$ km)Fig. 10 Simulation results ($Ma=0.5$, $H=0$ km)

put normalized W_f , while the rests of Fig. 7 are some primary quantities including the normalized static pressure of high compressor outlet p_{3s} , normalized thrust F , normalized π , normalized T_{ee} , n_{1c} and n_{2c} . Every quantity is related to two subfigures. The subfigure above contains three types of curves, the solid line means result of nonlinear model, the circle mark means result of the existing PLM and the cross mark means result of the improved PLM. The subfigure below shows relative errors, the solid line means the error between nonlinear model and the existing PLM, while the plus mark means the error between nonlinear model and the improved PLM. It can be seen from Fig. 7 that there is no difference between the existing PLM and the improved PLM, which is because there is no need to use the reduced parameters under $H=0$ km, $Ma=0$. In fact, the improved PLM is now simplified as the existing PLM.

Figure 8 illustrates the simulation results under $H=11$ km, $Ma=0.4$, just the same form as in Fig. 7. Differences between the existing PLM and the improved PLM can be seen in Fig. 8, especially the thrust F and the π . It seems that the improved PLM performs much better than the existing PLM, which confirms the analysis in section “the existing PLM”.

Figure 9 illustrates the simulation results under $H=11$ km, $Ma=0.8$. Apparently, the improved PLM shows better performance than the existing PLM.

Figure 10 illustrates the simulation results under $H=0$ km, $Ma=0.5$. Obviously, the improved PLM shows better performance than the existing PLM.

The simulation results shows

1) Under the standard sea-level condition, the existing PLM is just the same as the improved PLM, as stated before. They both perform well, the agreement of main parameters between linear and nonlinear models is within 4%.

2) Under the non-standard condition, the existing PLM shows poor performance. Although the agreement of T_{ee} and p_{3s} is within

5%, other parameters are far away from the nonlinear model. Conversely, it can be seen that the linear model using partition method perform well. The agreement of main parameters between improved PLM and nonlinear model is within 8%.

The conditions tested is very near to the boundary of flight envelope, that is to say, the improved PLM can be used in full area of flight envelope.

5 Conclusions and further development

In the testing process of PLM, it has been found that some reduced parameters won't meet the accuracy requirement when the operating condition is far away from the standard sea-level condition. After analyzing the usage of the similarity principle, it is not appropriate to use the reduced parameters. In order to avoid this abuse, a partition method has been proposed to improve the existing PLM.

This paper has introduced the partition method as well as the way to scheduling linear models, by which an improved turbofan PLM has been finished. After testing the PLM, the simulation results show that the improved PLM is the same as the existing PLM under standard sea-level condition. In terms of non-standard conditions, the agreement of parameter thrust between improved PLM and nonlinear model is within 8%, while between existing PLM and nonlinear model can be over 50% (thrust is the most inaccurate parameter). The validation results indicate that the improved PLM is much more accurate than existing PLM under the non-standard conditions. On the other hand, the computing speed of the improved PLM is much faster than nonlinear model.

This study does not address the following problem:

1) Smooth the output of model when the inner linear system switched.

2) Take different partition plans into consideration, and analyze the impact on the model.

These would be areas for further investigations.

Acknowledgments

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